

## ON A CONJECTURE OF H. WU

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In an email (January, 2015), Sullivan has asked the author how much one can do for coverings of compact complex manifolds. In this short note, we derive a conjecture of H. Wu from the theorem in [4], provided the fundamental group  $\pi_1(X)$  is residually finite. The author is grateful to Dennis Sullivan for the question.

**Conjecture (H. Wu).** *The universal covering  $U$  of a compact Kahler manifold  $X$  of dimension  $n$  with negative sectional curvature is a bounded domain in  $\mathbf{C}^n$ .*

**Proposition.** *Let  $U$  be the universal covering of a compact Kahler manifold  $X$  of dimension  $n$  with negative sectional curvature and residually finite  $\pi_1(X)$ . Then  $U$  is a bounded domain in  $\mathbf{C}^n$ .*

*Proof.* We consider  $U$  with the Kahler metric induced from  $X$ .

According to Wu [5] (see also [2, Problem B(ii), p. 45]),  $U$  is Stein hence  $\pi_1(X)$  is large, i.e.,  $U$  contains no proper submanifolds of positive dimension.

According to Ballmann and Eberlein [1],  $\pi_1(X)$  is nonamenable.

We will show  $X$  is projective. According to Gromov,  $X$  is Kahler hyperbolic [3, (0.3.A), p. 265]. The canonical bundle  $\mathcal{K}_X$  is quasiample [3, (0.4.C), p. 267; (3.2.B), (3.2.B') p. 287]. Thus  $X$  is Moishezon and Kahler hyperbolic hence  $X$  is projective. The same follows from a theorem of Kodaira as follows. Since the sectional curvature of  $X$  is negative, the Ricci form of the volume form is negative. Hence  $\mathcal{K}_X$  is ample by Kodaira.

Now, we can apply the theorem in [4, Introduction].

## REFERENCES

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